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Consequently,  $SE$  and  $SF$  are the projections of  $SB$  and  $SA$ , respectively, on the plane  $DSC$ .

Therefore  $\angle ESC < \angle BSC$ , and  $\angle FSC < \angle ASC$ ; or  $\angle DSC - \angle DSE < \angle BSC$ , and  $\angle DSC + \angle DSF < \angle ASC$ .

Adding, and remembering that  $\angle DSE = \angle DSF$ ,  $2 \angle DSC < \angle BSC + \angle ASC$ ,  $\angle DSC < \frac{1}{2}(\angle BSC + \angle ASC)$ .

CASE II.  $\angle DSC =$  a right angle. (See Fig. 2.)

To prove  $\angle DSC = \frac{1}{2}(\angle ASC + \angle BSC)$ .

Draw  $DX$  parallel to  $SC$ , and  $AB$  perpendicular to  $SD$ .

Then  $SD$  is perpendicular to  $DX$ , and, hence, to the plane determined by  $AB$  and  $DX$ .

This plane intersects the planes of faces  $BSC$  and  $ASC$  in  $BE$  and  $AF$ , respectively.

Since  $SC$  is parallel to  $DX$ , it is parallel to the plane  $BDX$ , and, hence, parallel to  $BE$  and  $AF$ .

Through  $SD$  pass a plane perpendicular to  $SC$ , intersecting the plane of  $AB$  and  $DX$  in  $EF$ , and the planes of faces  $BSC$  and  $ASC$  in  $SE$  and  $SF$ , respectively. Since  $CS$  is perpendicular to the plane  $FSE$ , so are  $BE$  and  $AF$ .

Hence,  $\angle$ 's  $BES$ ,  $BED$ ,  $AFS$ , and  $AFD$  are right angles.

Right triangles  $DAF$  and  $DBE$  are equal, since  $AD = BD$  (from equality of right triangles  $ASD$  and  $BSD$ ), and the vertical angles at  $D$  are equal.

Therefore  $BE = AF$ . Hence, since  $SB = SA$ , right triangles  $SEB$  and  $SFA$  are equal, and  $\angle ASF (= \angle ASH) = \angle BSE$ .

Now, since  $\angle$ 's  $CSD$ ,  $CSE$ , and  $CSH$  are right angles,  $\angle CSD = \frac{1}{2}(\angle CSE + \angle SCH)$ ,  $= \frac{1}{2}(\angle CSB + \angle BSE + \angle CSA - \angle ASH)$   $= \frac{1}{2}(\angle CSB + \angle CSA)$ .

CASE III.  $\angle DSC >$  a right angle. (See Fig. 3.)

To prove  $\angle DSC > \frac{1}{2}(\angle CSA + \angle CSB)$ .

Produce  $AS$ ,  $BS$ , and  $DS$ , forming another trihedral angle  $S - A'B'C$ .

By Case I,  $\angle CSD' < \frac{1}{2}(\angle CSA' + \angle CSB')$ , or  $180^\circ - \angle CSD < \frac{1}{2}(180^\circ - \angle CSA + 180^\circ - \angle CSB)$ , from which  $-\angle CSD < -\frac{1}{2}(\angle CSA + \angle CSB)$ .

Therefore,  $\angle CSD > \frac{1}{2}(\angle CSA + \angle CSB)$ .

Also solved by *H. C. WHITAKER*.

155. Proposed by *J. C. NAGLE, M.A., C.E.*, Professor of Civil Engineering, State Agricultural and Mechanical College, College Station, Texas.

A special case of the following problem was sent me some time ago by an ex-member of one of my engineering classes, as occurring on the Southern Pacific Ry., near Devil's River:

Two straight tracks,  $p$  feet between centers, are to be united by a cross-over composed of two curves of radius  $R$ , and a length  $L$  of intervening tangent. Required the central angles and the distance between tangent points, measured along main track. In the special case referred to  $p$  was 62 feet,  $L$  100 feet with  $9^\circ 30'$  curves.

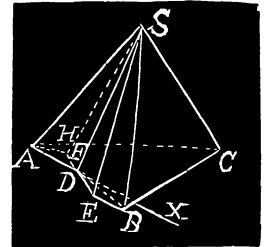


Fig. 2.

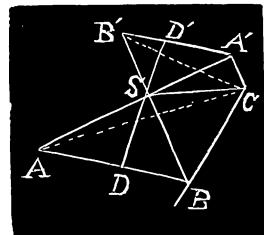


Fig. 3.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let  $\tan OCD = \tan EC'G = m$ ,  $OC = CD = C'G = C'E = R$ ,  $DE = L$ ,  $KO = LG = p$ . The equation to the line  $AH$  is  $y = mx - b$ , where  $O$  is origin and  $b = OA$ .  $CA = CD \sec OCD$ .  $\therefore R + b = R\sqrt{1 + m^2}$ .

$$\therefore y = mx + R - R\sqrt{1 + m^2}.$$

$$\text{When } y = 0, x = R[\sqrt{1 + m^2} - 1]/m,$$

$$\text{When } y = p, x = [p + R\sqrt{1 + m^2} - R]/m = x_1.$$

$$\therefore OB = BD = EF = FG = R[\sqrt{1 + m^2} - 1]/m.$$

$$\text{Since } BF = \sqrt{BT^2 + TF^2} = \sqrt{(x_1 - x)^2 + p^2},$$

$$\therefore L + 2R[\sqrt{1 + m^2} - 1]/m = \sqrt{(p/m)^2 + p^2} = (p/m)\sqrt{1 + m^2}.$$

$$\therefore [(2R - p)^2 - L^2]m^2 + 4RLm = 4Rp - p^2. \text{ This determines } m.$$

$$OL = OT + TL = x + x_1 = [p + 2R\sqrt{1 + m^2} - 2R]/m.$$

$$BR = BD \cos OCD = x/\sqrt{1 + m^2} = R[\sqrt{1 + m^2} - 1]/[m\sqrt{1 + m^2}].$$

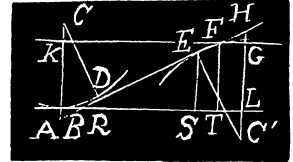
$$RS = BT - 2BR = p/m - 2R[\sqrt{1 + m^2} - 1]/[m\sqrt{1 + m^2}].$$

$$p = 62, L = 100, R = 604.$$

$$\therefore 325829m^2 + 60400m = 36487 \text{ or } m = .254559.$$

$$\therefore \angle OCD = 14^\circ 16' 52.6'' = \text{central angles.}$$

$$OL = 394.9 \text{ feet, } RS = 96.9 \text{ feet.}$$



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**CALCULUS.**  
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112. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A sphere of radius  $r$  is pierced by a cylinder radius  $\frac{1}{2}r$  so that the cylinder just grazes the center of the sphere. Find volume removed; the lateral surface and the spherical surface removed.

Solution by L. C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and the PROPOSER.

Taking the center of the sphere for coördinates, we have for the equations of sphere and cylinder, respectively,

$$x^2 + y^2 + z^2 = r^2, \quad y^2 = rx - x^2.$$

$$\text{Therefore volume removed, } V = 4 \int_0^r dx \int_0^{\sqrt{rx-x^2}} z dy$$

$$= 2 \int_0^r \left[ (r^2 - x^2) \sin^{-1} \sqrt{\frac{x}{r+x}} dx + \sqrt{(rx)(r-x)} dx \right] = \frac{3\pi - 4}{9} \cdot 2r^3.$$

The lateral surface  $S = 2 \int z ds$ ,  $s$  being an arc of the base circle of the cylinder. From  $y^2 = rx - x^2$ ,